

After bulky brane inflation

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Reheating or entropy production on the three-brane from decaying bulk scalar field is studied in the brane-world picture of the Universe. It is shown that a significant amount of dark radiation is produced in this process, so that subsequent entropy production within the brane is required before primordial nucleosynthesis.

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Any new theory of gravity and/or high energy physics must pass a number of cosmological tests, among which is implementation of cosmological inflation [1,2]. The successful inflation consists of two parts: sufficiently long quasi-exponential expansion driven by vacuum-like energy density such as a potential energy of a scalar field, and termination of accelerated expansion associated with entropy production or reheating to set the initial state of the classical hot Big Bang cosmology well before the primordial nucleosynthesis [3]. It is much more difficult to achieve the second element in general.

In this letter we consider reheating after inflation in the brane world picture of the Universe [4,5]. In this scenario, our Universe is described on the four-dimensional boundary (three-brane) of Z_2 -symmetric five-dimensional spacetime with a negative cosmological constant $\Lambda_5 \equiv -6k^2$, where k is a positive constant. This situation not only takes into account the spirit of Horava-Witten theory [6,7], but also recovers the Einstein gravity around the brane with positive tension [5,8,9]. Much work has been done on brane-world cosmology [10–12] including inflationary brane solutions [13–16].

We assume the five-dimensional Einstein gravity with a negative cosmological constant Λ_5 and a three-brane at the fifth coordinate $w = 0$ about which the spacetime is Z_2 symmetric. We write the metric near the brane in the following form in terms of the Gaussian normal coordinate.

$$ds_5^2 = g_{AB}dx^A dx^B = -N^2(t, w)dt^2 + Q^2(t, w)a^2(t)(dx^2 + dy^2 + dz^2) + dw^2 \equiv q_{\mu\nu}dx^\mu dx^\nu + dw^2, \quad (1)$$

where capital Latin indices run 0,1,2,3, and 5 while Greek indices from 0 to 3. We take $N = Q = 1$ on the brane $w = 0$. The explicit functional forms of $N(t, w)$ and $Q(t, w)$ are given in [12] in the case the bulk is in a vacuum state with Λ_5 . In this solution the induced metric on the brane is nothing but the spatially flat Robertson-Walker metric with the scale factor $a(t)$.

The evolution equation on the brane in this case is given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa_5^4 \sigma}{18} \rho_{\text{tot}} + \frac{\Lambda_4}{3} + \frac{\kappa_5^4}{36} \rho_{\text{tot}}^2 - \frac{k^2 C}{a^4}, \quad (2)$$

$$\Lambda_4 \equiv \frac{1}{2} \left(\Lambda_5 + \frac{\kappa_5^2}{6} \sigma^2 \right), \quad (3)$$

where κ_5^2 is the five dimensional gravitational constant related with the five dimensional reduced Planck scale, M_5 , by $\kappa_5^2 = M_5^{-3}$. σ is the brane tension and ρ_{tot} is the total energy density on the brane [9,11,12]. In order to recover the standard Friedmann equation with a vanishing cosmological constant at low energy scales, we require $\sigma = 6k/\kappa_5^2$ and $\kappa_4^2 = \kappa_5^4 \sigma / 6 = \kappa_5^2 k$, where κ_4^2 is the four dimensional gravitational constant related with the four dimensional reduced Planck scale, M_4 , as $\kappa_4^2 = M_4^{-2}$. We therefore find $M_4^2 = M_5^3/k$. That is, if we take $k = M_4$, all the fundamental scales in the theory take the same value, $k = M_4 = M_5$. Note that k also sets the scale above which the nonstandard term quadratic in ρ_{tot} is effective in (2). We assume that k is much larger than the scale of inflation so that such quadratic corrections are negligible.

We consider the case inflation is driven by a bulk scalar field ϕ with a five-dimensional potential $V[\phi]$ [15,16] and study the evolution of ϕ after brane inflation, because reheating is expected to proceed in the same way as in four dimensional theory if the inflaton lives only on the brane [14]. Since ϕ is homogeneized in three space as a result of inflation, it depends only on t and w . We consider a situation that ϕ rapidly oscillates around $\phi = 0$ with $V[\phi]$. Then the Klein-Gordon equation reads

$$\square_5 \phi(w, t) - V'[\phi(w, t)] = \frac{1}{\sqrt{-g}} \partial_t (\sqrt{-g} g^{00} \dot{\phi}) + \frac{1}{\sqrt{-g}} \partial_w (\sqrt{-g} \phi_{,w}) - V'[\phi] = 0, \quad (4)$$

where a dot denotes time differentiation. In order to express energy release from ϕ due to interaction with other fields we introduce following dissipation terms phenomenologically in the right-hand-side of (4).

$$\square_5 \phi(w, t) - V'[\phi(w, t)] = \frac{\Gamma_D}{2k} \delta(w) \frac{1}{N} \dot{\phi} + \Gamma_B \frac{1}{N} \dot{\phi}. \quad (5)$$

Here Γ_D and Γ_B represent energy release to the brane and to the entire space, respectively. The denominator in the Γ_D term is introduced on dimensional grounds.

From (4) and (5) together with the Z_2 symmetry, we find

$$\phi_{,w}^+ = -\phi_{,w}^- = \frac{\Gamma_D}{4k} \dot{\phi}(0, t), \quad (6)$$

where superscripts $+$ and $-$ imply values at $w \longrightarrow +0$ and -0 , respectively. The divergence of the energy-momentum tensor of the scalar field,

$$T_{MN}^{(\phi)} = \phi_{,M} \phi_{,N} - g_{MN} \left(\frac{1}{2} g^{PQ} \phi_{,P} \phi_{,Q} + \frac{1}{2} m^2 \phi^2 \right), \quad (7)$$

reads,

$$T_{A;C}^{(\phi)C} = \left\{ \square_5 \phi(w, t) - V'[\phi(w, t)] \right\} \phi_{,A} = \left[\frac{\Gamma_D}{2k} \delta(w) \frac{1}{N} \dot{\phi} + \Gamma_B \frac{1}{N} \dot{\phi} \right] \phi_{,A}. \quad (8)$$

Integrating $A = 0$ component of (8) from $w = -\epsilon$ to $w = +\epsilon$ near the brane, we find (6) from the zeroth order in ϵ and

$$\frac{\partial \rho_\phi(0, t)}{\partial t} = -(3H + \Gamma_B) [\rho_\phi(0, t) + p_\phi(0, t)] = -(3H + \Gamma_B) \dot{\phi}^2(0, t), \quad (9)$$

with

$$\rho_\phi(0, t) \equiv -T_0^0(0, t) = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \phi_{,w}^2 + V[\phi], \quad p_\phi(0, t) \equiv T_x^x(0, t) = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \phi_{,w}^2 - V[\phi], \quad (10)$$

from the terms proportional to ϵ . Thus the energy dissipated by the Γ_D term on the brane is entirely compensated by the energy flow onto the brane which is proportional to $\phi_{,w}$.

Next we study how the energy released from ϕ affects evolution of our brane Universe by analyzing gravitational field equations [9,16]. In the present situation the total energy momentum tensor including the contribution of bulk cosmological constant reads

$$T_{MN} = -\kappa_5^{-2} \Lambda_5 g_{MN} + T_{MN}^{(\phi)} + S_{MN} \delta(w), \quad (11)$$

where S_{MN} is the stress tensor on the brane. Its nonvanishing components can be further decomposed as

$$S_{\mu\nu} = -\sigma q_{\mu\nu} + \tau_{\mu\nu}. \quad (12)$$

Here $\tau_{\mu\nu}$ represents energy momentum tensor of the radiation fields produced by decay of ϕ and it is of the form $\tau_\nu^\mu = \text{diag}(-\rho_r, p_r, p_r, p_r)$ with $p_r = \rho_r/3$.

In terms of the unit vector $n_M = (0, 0, 0, 0, 1)$ normal to the brane, the extrinsic curvature of a $w = \text{constant}$ hypersurface is given by $K_{MN} = q_M^P q_N^Q n_{Q;P}$ with $q_{MN} = g_{MN} - n_M n_N$. Then from the Codazzi equation and the five dimensional Einstein equation, we find

$$D_\nu K_\mu^\nu - D_\mu K = \kappa_5^2 T_{MN} n^N q_\mu^M = \kappa_5^2 T_{\mu w} = \kappa_5^2 \dot{\phi} \phi_{,w} \delta_\mu^0, \quad (13)$$

where D_ν stands for the four dimensional covariant derivative with respect to the metric $q_{\mu\nu}$. The above equation reads

$$D_\nu K_0^{\nu+} - D_0 K^+ = \kappa_5^2 \frac{\Gamma_D}{4k} \dot{\phi}^2(0, t), \quad (14)$$

near the brane, $w \longrightarrow +0$.

From the junction condition and the Z_2 -symmetry, on the other hand, we find

$$K_{\mu\nu}^+ = -\frac{\kappa_5^2}{2} \left(S_{\mu\nu} - \frac{1}{3} q_{\mu\nu} S \right), \quad (15)$$

therefore

$$D_\nu K_\mu^{\nu+} - D_\mu K^+ = -\frac{\kappa_5^2}{2} D_\nu S_\mu^\nu = -\frac{\kappa_5^2}{2} D_\nu \tau_\mu^\nu. \quad (16)$$

Combining (14) and (16), we obtain

$$D_\nu \tau_\mu^\nu = -\frac{\Gamma_D}{2k} \dot{\phi}^2 \delta_\mu^0, \quad (17)$$

namely,

$$\frac{\partial \rho_r}{\partial t} = -3H(\rho_r + p_r) + \frac{\Gamma_D}{2k} \dot{\phi}^2 = -4H\rho_r + \frac{\Gamma_D}{2k} \dot{\phi}^2, \quad (18)$$

on the brane. Thus we find only the dissipation term proportional to Γ_D with the delta function is effective to reheat the brane. This equation has the same form as the reheating in perturbation theory after conventional inflation in four dimensional theory [17].

On the other hand, the four dimensional Einstein tensor, $G_{\mu}^{(4)\nu}$, satisfies the following equality on the brane [16],

$$G_{\mu}^{(4)\nu} = \kappa_4^2 (T_{\mu}^{(s)\nu} + \tau_{\mu}^{\nu}) + \kappa_5^4 \pi_{\mu}^{\nu} - E_{\mu}^{\nu}, \quad (19)$$

with

$$T_{\mu}^{(s)\nu} \equiv \frac{1}{6k} \left[4q^{\nu\zeta} \phi_{,\mu} \phi_{,\zeta} + \left(\frac{3}{2} \phi_{,w}^2 - \frac{5}{2} q^{\xi\zeta} \phi_{,\xi} \phi_{,\zeta} - \frac{3}{2} m^2 \phi^2 \right) q_{\mu}^{\nu} \right]. \quad (20)$$

Here π_{μ}^{ν} represents terms quadratic in τ_{α}^{β} which are higher order in $\rho_r/(kM_4)^2$ and are consistently neglected in our analysis. $E_{\mu}^{\nu} \equiv C_{\mu w}^{w\nu}$ is a component of the five dimensional Weyl tensor C_{PQ}^{MN} , which is the origin of the dark radiation [12].

Now we write down the four dimensional Bianchi identity,

$$D_{\nu} G_{\mu}^{(4)\nu} = 0 = \kappa_4^2 (D_{\nu} T_{\mu}^{(s)\nu} + D_{\nu} \tau_{\mu}^{\nu}) - D_{\nu} E_{\mu}^{\nu}, \quad (21)$$

to yield

$$D_{\nu} E_0^{\nu} = -\frac{\kappa_4^2}{2k} \frac{\partial}{\partial t} \left(\frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \phi_{,w}^2 + V[\phi] \right) - \frac{2\kappa_4^2 H}{k} \dot{\phi}^2 - \frac{\kappa_4^2}{2k} \Gamma_D \dot{\phi}^2. \quad (22)$$

Note that $\phi_{,w}^2 = \Gamma_D^2 \dot{\phi}^2/(16k^2)$ on the brane and is negligibly small compared with $\dot{\phi}^2$, since we expect the dissipation rate Γ_D is at most comparable to the scale of inflation and hence much smaller than the scale k . With this approximation the quantity in the parenthesis in (22) may be replaced by $\rho_{\phi}(0, t)$. We also find $D_{\nu} E_0^{\nu} = \partial_0 E_0^0 + 4H E_0^0$ because E_{μ}^{ν} is traceless.

As a result we obtain the following set of evolution equations in the brane universe $w = 0$ in terms of $\varphi(t) \equiv \phi(0, t)/\sqrt{2k}$.

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{\kappa_4^2}{3} (\rho_{\varphi} + \rho_r + \varepsilon), \quad \rho_{\varphi} \equiv \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2k} V[\sqrt{2k}\varphi], \quad \varepsilon \equiv \kappa_4^{-2} E_0^0, \quad (23)$$

$$\frac{\partial \rho_{\varphi}}{\partial t} = -(3H + \Gamma_B) \dot{\varphi}^2 \equiv -(3H + \Gamma_B)(1 + \omega_{\varphi}) \rho_{\varphi}, \quad (24)$$

$$\frac{\partial \rho_r}{\partial t} = -4H \rho_r + \Gamma_D \dot{\varphi}^2, \quad (25)$$

$$\frac{\partial \varepsilon}{\partial t} = -4H \varepsilon - (H - \Gamma_B) \dot{\varphi}^2 - \Gamma_D \dot{\varphi}^2, \quad (26)$$

where we have used (9). Although the above equations depend solely on quantities on the brane and they resemble those in the case of perturbative reheating in the ordinary inflation to this end [17], except of course for those involving ε , we cannot solve them as they are. This is because the effective equation of state of φ , which is characterized by ω_{φ} defined in (24), is in general different from that of dust even when the mass term dominates the potential, due to the presence of the extra dimension or Kalza-Klein modes [18]. In fact, ω_{φ} depends on the model of inflation as well as on the initial condition. Furthermore in general it changes in time.

Here we qualitatively analyze the evolution of the system in various cases after appropriate classification. We assume that both ρ_r and ε are vanishing at the end of inflation when we set the initial condition, because they rapidly redshift during inflation and very little creation is expected in that period [16]. We identify the reheating epoch with the time when ρ_{φ} becomes smaller than ρ_r .

Case A: The case Γ_B is vanishingly small and ϕ loses its energy by decaying into fields on the brane.

In this case, reheating is completed if ω_{φ} becomes large enough and ρ_{φ} redshifts more rapidly than radiation, which is realized if the energy of the bulk scalar field ϕ is exhausted through the energy flow onto the brane. (Otherwise there is no reheating and no recovery to the hot Big Bang Universe.) Since the magnitude of the creation term of dark radiation, $-(H + \Gamma_D) \dot{\varphi}^2$, is always larger than that of radiation, $\Gamma_D \dot{\varphi}^2$, more dark radiation is created than ordinary radiation in magnitude. Then ρ_r and ε tend to cancel each other and the higher-order terms of the Friedmann equation, which we have neglected so far, would play an important role in the subsequent evolution of the three brane. This means that we do not recover standard cosmology on the brane after inflation.

Case B: The case ω_{φ} remains smaller than 1/3 until ρ_{φ} decays due to the Γ_B term.

In this case reheating occurs when H becomes as small as Γ_B . Until this epoch total magnitude of the creation terms in (26) is larger than that of radiation, hence we again end up with more dark radiation than ordinary radiation in magnitude, unless the reheating epoch is delayed and fine tuned with $\omega_{\varphi} < 0$.

We therefore find in general we do not recover the standard hot Big Bang Universe after brane inflation if it is driven by a bulk scalar field ϕ and if the reheating on the brane proceeds as ϕ decays directly into radiation fields. In order to cure this problem, we must introduce some other mechanisms of entropy production before primordial nucleosynthesis which imposes stringent constraints on the exotic energy density and the expansion law at that era [3]. One way is to assume that ϕ predominantly decays into massive particles whose energy density redshifts less rapidly and dominates over ε soon. If they decay into radiation before nucleosynthesis creating an appropriate amount of baryon asymmetry, we may recover the

standard cosmology. Another way is to consider second inflation which is driven by a scalar field confined on the brane [14] to dilute ε .

In summary we have studied entropy production on the three-brane from a decaying bulk scalar field ϕ by introducing dissipation terms to its equation of motion phenomenologically. We have found only a dissipation term proportional to $\delta(w)$ is effective to produce entropy on the brane. We have also shown that the so-called dark radiation is inevitably produced at the same time which typically has negative energy density. Although we have analyzed only the case with a rather specific form of dissipation terms, we expect our conclusion is generic and applicable to other forms of dissipation, too, because it is essentially an outcome of the four dimensional Bianchi identity (21). We therefore conclude that in the brane-world picture of the Universe the dominant part of the entropy we observe today originates within the brane rather than in the bulk.

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